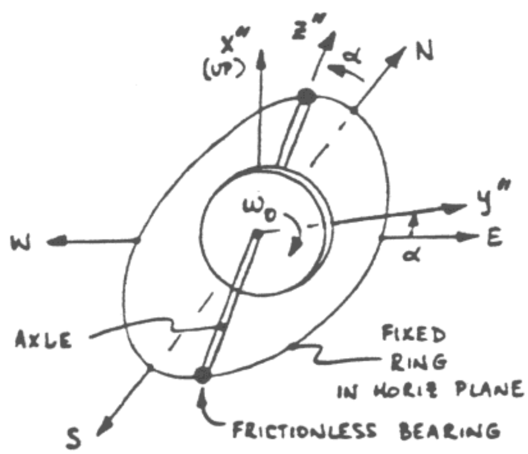
University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

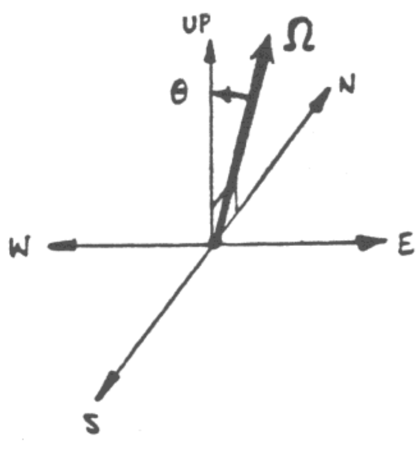
ASSIGNMENT 10

Reading:
105 Notes 12.1-12.4
Hand & Finch 9.1-9.6

1. and **2.** (double credit problem)
The *Foucault gyrocompass* is a gyroscope that eventually, taking advantage of frictional damping, points to true (not magnetic) north. Thus it is an essential guidance system component.

The gyrocompass may be modeled as a thin disk spinning with angular frequency ω_0 about its symmetry axis z''. This axis can move freely in the horizontal (North-South-East-West) plane only. As exhibited in the following diagrams, the z'' axis makes an angle $\alpha(t)$ with North. The gyrocompass is located at colatitude θ on an earth spinning with angular frequency Ω.





Assuming that $\omega_0 \gg \Omega$ and $\omega_0 \gg \dot{\alpha}$, prove that $\alpha(t)$ oscillates about $\alpha = 0$ provided that $\alpha \ll 1$. Find the angular frequency of oscillation. Note that friction in the bearings will eventually damp out this oscillation, enabling the gyrocompass to point to true north, as defined by the earth's axis of rotation.

You may find the following hints useful:

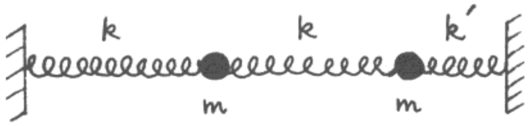
- Work the problem in the body ($''$) system. This system is obviously not the same as the fixed ($'$) system. It is also not the same as the unprimed system, which is the North-South-East-West system attached to the earth. Using Euler's equations would require knowing the torque from the bearings, evaluated in the body system. Since this torque is not known *a priori*, Euler's equations are not useful here.

- Write $\omega_{x''}$, $\omega_{y''}$, and $\omega_{z''}$ in terms of Ω, α, $\dot{\alpha}$, and θ.

- To get the relationship between the torque $\mathbf{N}'$ applied by the bearings and the angular momentum $\mathbf{L}''$, first write $\mathbf{N}' = d\mathbf{L}'/dt$ (taking advantage of the fact that the ($'$) system is inertial.) Then transform $\mathbf{L}'$ to the $''$ system.

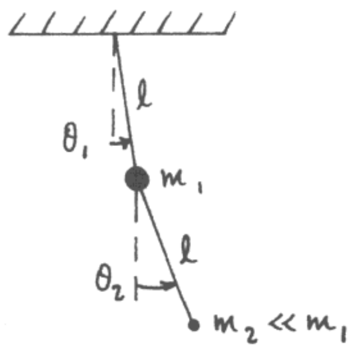
- When evaluating $\mathbf{L}$, remember to neglect terms that are smaller by a factor Ω/ω_0 than the leading terms.

3.
Consider a coupled oscillator with two equal masses m, each connected to fixed supports by springs with unequal spring constants k and k'. The two masses are connected to each other by a spring with spring constant k.



Find its two natural angular frequencies.

4.
Consider a double pendulum as exhibited in the following diagram. The two pendula are of equal lengths ℓ, but the lower mass $m_2 \ll m_1$. Choose θ_1 and θ_2, the angles between each string and the vertical, as generalized coordinates.

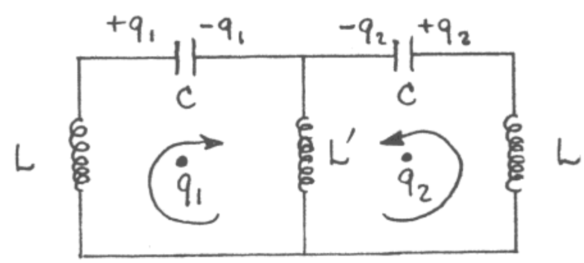


(**a**)
Find the natural angular frequencies of oscillation.
(**b**)
Calculate the interval $\mathcal{T}/2$ between times for which one or the other bob has minimum amplitude of oscillation. [*Hint:* This is $\pi/\Delta\omega$, where $\Delta\omega$ is the difference between the two natural angular frequencies.]

5.
Consider a linear triatomic molecule, as in the diagram below. A mass M is connected to two masses m, one on either side, by springs of equal spring constant k.

(**a**)
Find the three natural frequencies of the linear triatomic molecule.
(**b**)
One of these frequencies should be zero. To what motion does it correspond?

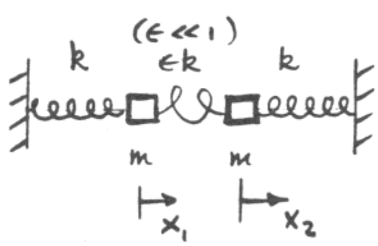
6.
In a series LC circuit, choose the charge q and its first derivative $\dot{q}$ as independent variables. Equate the "kinetic energy" T to $\frac{1}{2}L\dot{q}^2$ and the "potential energy" U to $\frac{1}{2}q^2/C$. Then Lagrange's equations produce the usual differential equation for the circuit.



In analogy with this approach, find the resonant frequencies of the above LC circuit. Do not rely on loop equations or any other circuit theory. Instead, write the analogous circuit Lagrangian and solve formally using coupled oscillator methods.

7.
Consider a thin homogeneous plate of mass M which lies in the $x_1 - x_2$ plane with its center at the origin. Let the length of the plate be $2A$ (in the x_2 direction) and let the width be $2B$ (in the x_1 direction). The plate is suspended from a fixed support by four springs of equal force constant k located at the four corners of the plate. The plate is free to oscillate, but with the constraint that its center must remain on the x_3 axis. Thus, there are 3 degrees of freedom: (1) vertical motion, with the center of the plate moving along the x_3 axis; (2) a tipping motion lengthwise, with the x_1 axis serving as an axis of rotation (choose an angle θ to describe this motion); and (3) a tipping motion sidewise, with the x_2 axis serving as an axis of rotation (choose an angle ϕ to describe this motion).
(**a**)
Assume only small oscillations and show that the secular equation has a double root and, hence, that the system is degenerate.
(**b**)
Discuss the normal modes of the system.
(**c**)
Show that the degeneracy can be removed by the addition to the plate of a thin bar of mass m and length $2A$ which is situated (at equilibrium) along the x_2 axis. Find the new eigenfrequencies of the system.

8.
Consider a pair of equal masses m connected to walls by equal springs with spring constant k. The two masses are connected to each other by a much weaker spring with spring constant ϵk, where $\epsilon \ll 1$. Choose x_1 and x_2, the displacements from equilibrium of the two masses, as the generalized coordinates.



For this system, write...
(**a**)
...the spring constant matrix $\mathcal{K}$ and the mass matrix $\mathcal{M}$
(**b**)
...the normal frequencies ω_1 and ω_2
(**c**)
...the normal mode vectors $\tilde{a}_1$ and $\tilde{a}_2$ (corresponding to ω_1 and ω_2), each expressed as a linear combination of x_1 and x_2
(**d**)
...the 2×2 matrix $\mathcal{A}$ which reduces $\mathcal{M}$ to the unit matrix via the congruence transformation

$$\mathcal{I} = \mathcal{A}^t \mathcal{M} \mathcal{A} ,$$

where $\mathcal{I}$ is the identity matrix
(**e**)
...the normal coordinates Q_1 and Q_2 (corresponding to ω_1 and ω_2), each expressed as a linear combination of x_1 and x_2.